

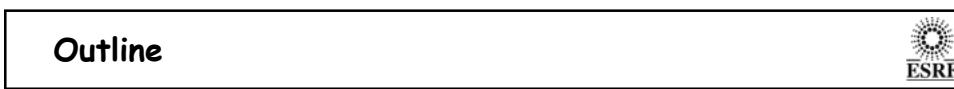
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Thermomechanical analysis for synchrotron radiation instrumentation

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Outline



- **Introduction**
 - Covered items, type of analyses, methods
- **Analytical analysis**
 - Equations, Boundary conditions, analytical solution, numerical solution, shell/beam theory, some examples
- **Finite Element Analysis**
 - Model generation, mesh, BC, post-processing, non-linear, transient, coupled (multi-field), results validation
- **Case study : X-ray mirror and monochromator crystal**
- **Some other applications**



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Introduction



- **Thermomechanical analysis plays roles in the**
 - Feasibility study
 - Early design stage, design optimization and simulation
 - Understanding of physics phenomena
- **Covered items**
 - **Machine components:** thermal absorbers, FE window and attenuator, vacuum chambers, girders, diagnostic mirror, ...
 - **Beamline components:** optics (mirror & monochromator crystal), refractive lens, bending device, slits, attenuators, tables, supports, vacuum vessels, ...
 - **Some other applications:** nano-structures, silicon wafer,...
- **Challenges**
 - Extreme high heat load ($\sim 100 \text{ W/mm}^2$)
 - High precision and stability requirements (better than $1 \mu\text{m}$)
 - Ultra-high vacuum ($\sim 10^{-10} \text{ mbar}$)

Introduction - cndt**➤ Type of analysis**

- Thermal, mechanical (static, dynamic)
- Coupled : *thermomechanical, piezoelectrical, electromagnetical, ...*
- (linear, steady-state)
- Non-linear : *temperature-dependent material property, piezoelectricity, material non-linearity, geometric non-linearity (large deformation, stress stiffening,...), contact, ...*
- Transient : *sudden condition change, periodic condition change,...*

➤ Methods

- Analytical : *equation integration, formulation (Roark's,...), empirical formulation, tables,...*
- Numerical : *equation solving, Finite-Element Analysis (FEA)*

➤ Validation

- By experiments, analytical/FEA

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➤ Finite Element Analysis

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➤ Case study : X-ray mirror and monochromator crystal**➤ Some other applications**

Analytical analysis



- ➤ **Solid mechanics**
- **Fluid mechanics (isothermal fluids)**
- **Heat transfer**
 - Heat transfer in solids (thermal conduction, radiation)
 - Heat transfer in fluids (fluid mechanics in non-isothermal fluids)
- **Coupling**
 - Mechanical → thermal: very weak, negligible
 - Thermal → mechanical: strong, possibly dominant
 - Other coupling
- **Thermal, mechanical, thermomechanical analysis**

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Heat transfer - equations



- Energy conservation → temperature gradient, heat source/sink, temperature rate ($\partial T/\partial t$)

$$\operatorname{div}\left(k * \overrightarrow{\operatorname{grad}(T)}\right) + q = \rho c_p \frac{\partial T}{\partial t}$$

for a homogeneous and isotropic solid

- Linear or non-linear material properties: k , ρ , c_p
- Heat source or sink q , for instance:
 - high energy photons absorbed by bulk material
 - thermal radiation
- Steady-state ($\partial T/\partial t=0$), transient ($\partial T/\partial t \neq 0$)

Heat transfer - boundary conditions



➤ Heat transfer modes

- Conduction: $k^* \text{grad}(T)$
- Convection: $h^*(T_b - T_f)$
- Radiation: $\varepsilon * \sigma^*(T_1^4 - T_2^4)$

➤ Boundary conditions

- Isothermal: $T_{b-i} = \text{constant-}i, i=1, 2, \dots$
- Isolating: $\text{grad}(T_{b-\nu}) \cdot n = 0, i=1, 2, \dots$
- Convection: $k^* \text{grad}(T_{b-\nu}) \cdot n = h^*(T_{b-i} - T_f)$
- Combined

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Heat transfer - an example of 2D temperature distribution



➤ Top surface heat load + bottom convection cooling

- Bending magnet thermal absorber
- Small grazing angle mirror

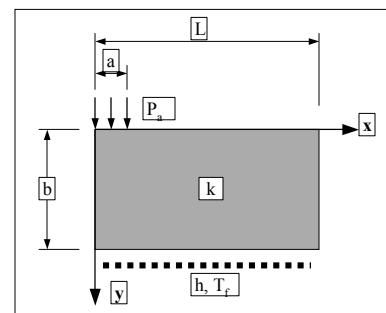
➤ Equation and boundary conditions

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x=0, x=L$$

$$\frac{\partial T}{\partial y} = -\frac{P_a}{k} f(x) \quad \text{at } y=0$$

$$\frac{\partial T}{\partial y} = -\frac{h}{k} (T - T_f) \quad \text{at } y=b$$



$$f(x) = \begin{cases} 1 & \text{for } x \leq a \\ 0 & \text{for } a < x \leq L \end{cases}$$

Heat transfer - 2D temperature distribution (ctd)

> Analytical solution

$$T(x, y) = T_f + \frac{aP_a}{kL} \left(\frac{k}{h} + b - y \right) + \frac{2P_a}{kL} \sum_{n=0}^{\infty} \frac{\cosh(\beta_n(b-y)) + \gamma_n \sinh(\beta_n(b-y)) * \sin(\beta_n a)}{\sinh(\beta_n b) + \gamma_n \cosh(\beta_n b)} \cos(\beta_n x)$$

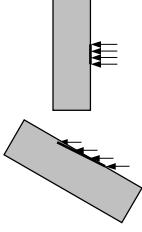
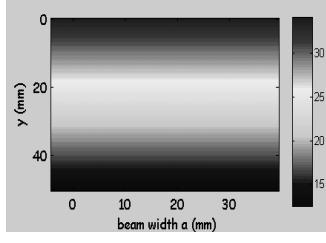
with $\beta_n = n\pi/L$ $\gamma_n = h/k\beta_n$

> Temperature averaged over x $\frac{1}{L} \int_0^L (\dots) dx$: \downarrow $aP_a = \text{cte}$

$$\frac{1}{L} \int_0^L \cos(\beta_n x) dx \equiv 0$$

$T_{mean}(y) = T_f + \frac{aP_a}{kL} \left(\frac{k}{h} + b - y \right)$

$\frac{dT_{mean}}{dy} = -\frac{aP_a}{kL}$

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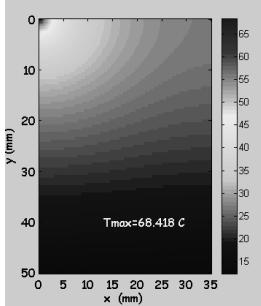
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Heat transfer - 2D temperature distribution (ctd)

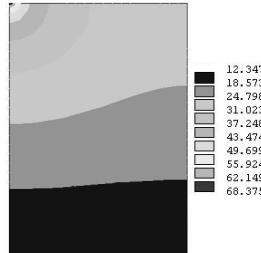
> Comparison with FEA results

- $L=35$ mm
- $b=50$ mm
- $a=0.75$ mm
- $P_a=2.917 \text{ W/mm}^2$
- $h=0.005 \text{ W/mm}^2/\text{C}$, $T_f=0 \text{ C}$
- $k=0.148 \text{ W/mm/C}$
- $T_{max-ANSYS}=68.375 \text{ C}$
- $T_{max-analytic}=68.418 \text{ C}$
- $\Delta = -0.6\%$

• Analytical by MatLab



• FEA by ANSYS



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Mechanics, thermomechanics



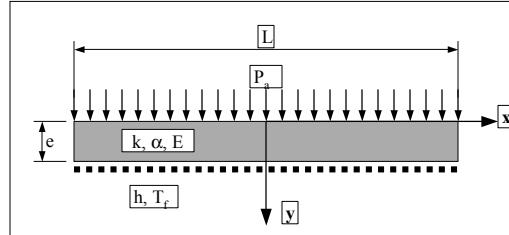
- Many terms in mechanics
 - Stress, strain tensors, displacement
 - Tension, compression, torsion, bending, shearing
 - Hooke's law (anisotropic, isotropic, orthotropic...)
 - Elasticity, plasticity, creep (time-dependent deformation)
 - Stress concentration, fracture mechanics, fatigue
 - Yield strength, fatigue strength, ultimate strength
 - ...
- Model of beam, shell, plate, membrane...
 - Roark's, Timoshenko, empirical formulation, tables, ...,
- Thermal strain: $\epsilon_{th} = \alpha T$
- Analyses
 - Static: stress, deformation
 - Dynamic or stability: buckling, modal (natural frequencies, modes), frequency response analysis

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Thermomechanics - an example of thermal bending beam



A long mirror with uniform power P_a on the top surface and cooled on the bottom surface, constant material properties



- Linear temperature distribution: $T(y) = T_f + \frac{P_a}{k} \left(\frac{k}{h} + e - y \right)$ $\Delta T = \frac{e P_a}{k}$

- Bending radius of curvature: $\frac{1}{R} = \frac{M_T}{EI}$

with $M_T = \int_0^e \alpha ET(y)(y - e/2)dy$

$\frac{1}{R} = \frac{\alpha P_a}{k} = \frac{\alpha \Delta T}{e}$



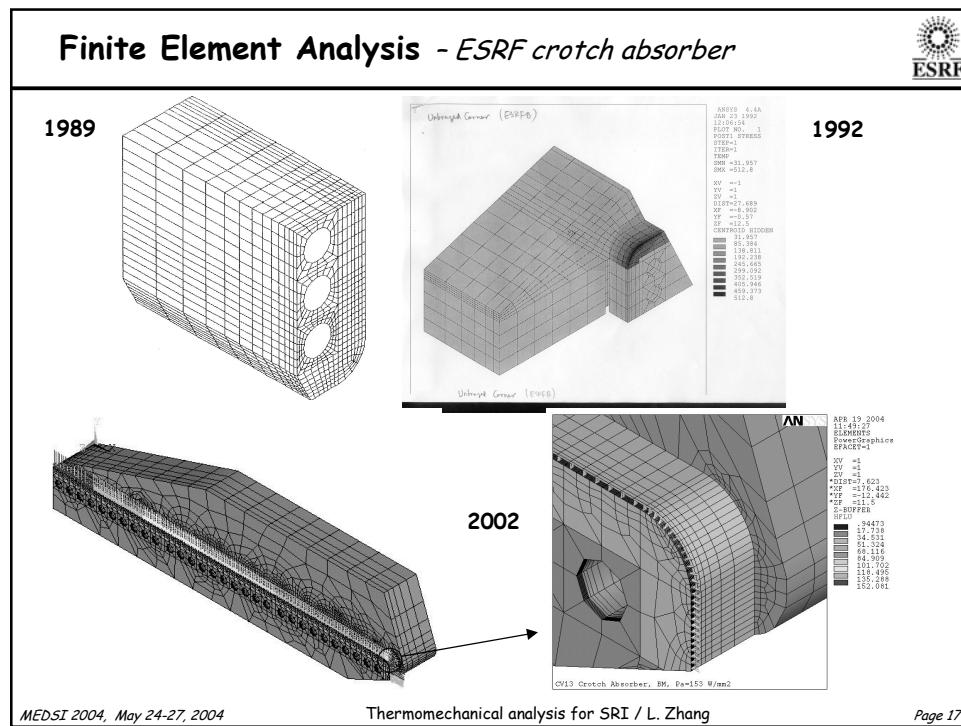
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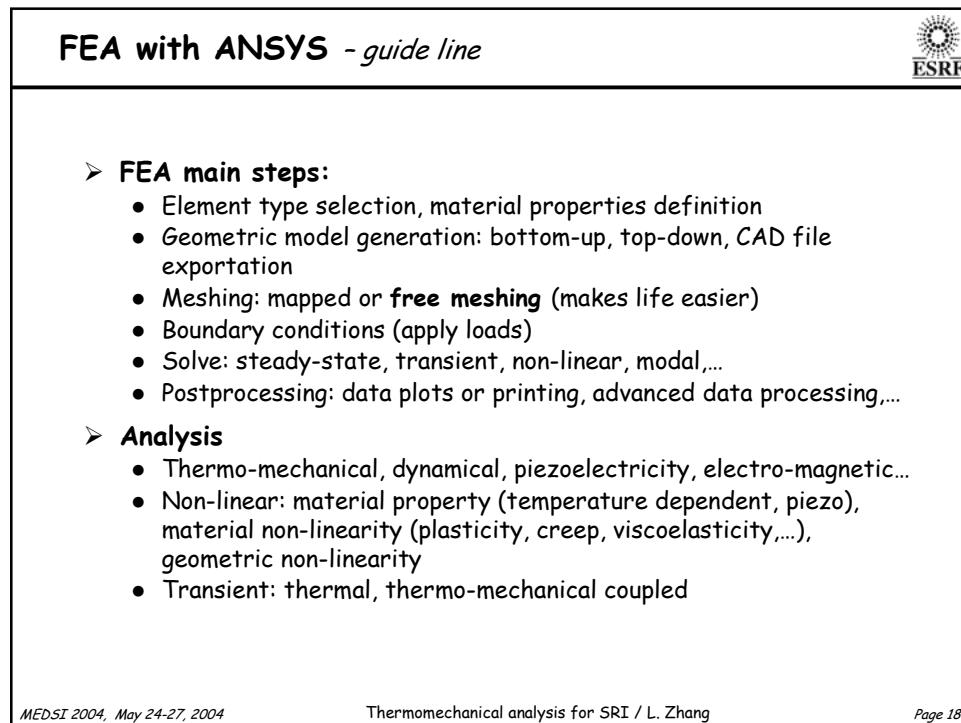
Finite Element Analysis



- **Many commercial software packages**
 - ANSYS, COSMOS used at the ESRF
 - NASTRAN, IDEAS, ABAQUS, ...
 - Bonded FEA with CAD: Designspace, Cosmosworks, ...
 - FEMLAB : based on MatLab, can easily solve user defined equations
- **Powerful tools**
 - 15 years ago / today
 - disk space 500 Mb / 100 Gb
 - Nb of elements ~ 2000 / 200 000
 - Meshing techniques: free meshing, transition of different shapes of elements
 - Solvers: more robust and faster
 - CAD file import for geometry generation
 - Tighter connection between CAD and FEA
 - Fast CAE processes
 - Complex geometry possible
 - Compatibility issues (parasolid v14, igs used at the ESRF,...)



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FEA validation

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- **FEA validation**
 - By physics: parameter dependence, order of magnitude
 - By comparison with analytical estimation
 - By comparison with experiment results
- **Some hints**
 - Units consistency (material properties and geometry)
 - Geometry connection of different parts, and meshing continuation
 - Boundary conditions (!!) checking by
 - Power balance
 - Force balance (reaction force,...)
 - Parameter sensibility or dependency:
 - $\Delta T \sim P/K$, $\Delta\theta \sim \alpha P/k$, $\sigma \sim E$, $\varepsilon \sim \alpha$, ...
 - in a bended (thin) mirror $\sigma \sim 1/t^2$ for $M=cte$, $\sigma \sim t$ for $R=cte$
 - Different element types and meshing → convergent results

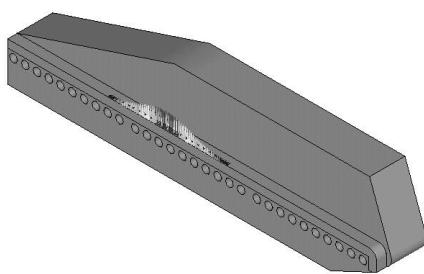
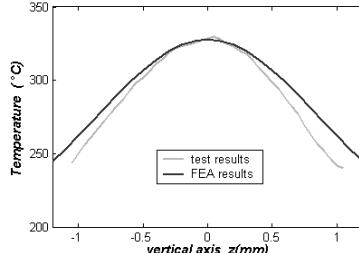
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FEA validation- crotch absorber

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- **Crotch absorber tested with 2U34 undulators**
 - $P_{a_{max}} = 675 \text{ W/mm}^2$
 - $\alpha_{inc} = 7.5^\circ$
 - $P_{a_{max-projected}} = 88 \text{ W/mm}^2$
 - $h = 0.02 \text{ W/mm}^2/C$, $T_{water} = 22^\circ\text{C}$
- **Temperature comparison between Infrared measurements and FEA results**

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X-ray mirror and monochromator crystal



<i>Optics</i>	<i>Typical dimension</i>	<i>Grazing (Bragg) angle</i>	<i>Material (of substrate)</i>	<i>Optical passband</i>
Mirror	1000 × 50 × 50	~ mrad	Si, SiC, ...	Low-pass
Monochromator	100 × 50 × 50	~ degrees	Si, Diamond,...	Narrow passband
Multilayer	Between mirror and mono		Si, ...	Wide passband

- **Mirror**
 - Long footprint, small power density
 - Mostly cooled by water
- **Monochromator**
 - Short footprint, high power density
 - Cooled by LN2 (liquid nitrogen), He gas at T_{LN2} , water

Total power	500	W
beam size (HxV)	2 × 1	mm ²
	mirror	mono
incident angle	2.5 mrad	14°
footprint (mm)	2 × 400	2 × 4.13
Pa (W/mm ²)	0.625	60.5

Thermal deformation - theoretical analysis

ESRF

➤ **Two components**

- **Bending**: due to the temperature difference ΔT between the upper and lower surfaces. This ΔT is caused by the fact of heat flux on the upper surface and cooling from the bottom
- **Bump**: due to the variation of the thickness in the x-direction. This thickness variation is the consequence of the temperature or heat flux variation along the x-axis

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Thermal deformation - theoretical analysis (2)

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From the theory of the (mechanical) beam:

$$\frac{M_{Tx}}{EI_x} = \frac{1}{R} \cong \frac{\partial^2 U_y}{\partial z^2} = \frac{\partial \theta(z)}{\partial z}$$

$$M_{Tx} = \iint_A \alpha ET(x, y, z) y dA \quad I_x = \iint_A y^2 dA$$

$$\theta(z) = \frac{12}{wt^3} \int_0^{z-t/2} \int_{-w/2}^{w/2} \int \alpha T(x, y, z) dx dy dz$$

In some cases and constant material properties:

$$\Delta \theta = \frac{\alpha P}{k w} f(t, L, Bi, L_{bm}) \quad \Delta \theta \propto \frac{\alpha P}{k w}$$

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Thermal deformation - theoretical analysis (3)

Uniform heat flux on the top surface, cooled on the bottom surface,

$$T(x, y, z) = T_m + \frac{yP}{wLk}$$

$$\theta = \frac{z}{R_0} \quad \frac{1}{R_0} = \frac{\alpha}{k} \frac{P}{wL}$$

→ Pure bending

$\theta(z) = \frac{12\alpha}{wt^3} \int_{0-t/2}^{t/2} \left[\int_{-w/2}^{w/2} T(x, y, z) dx \right] y dy dz$

$\int_{-w/2}^{w/2} T(x, y, z) dx = f(z) \rightarrow \theta(z) = 0$

Side cooling

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Thermal deformation - theoretical analysis (4)

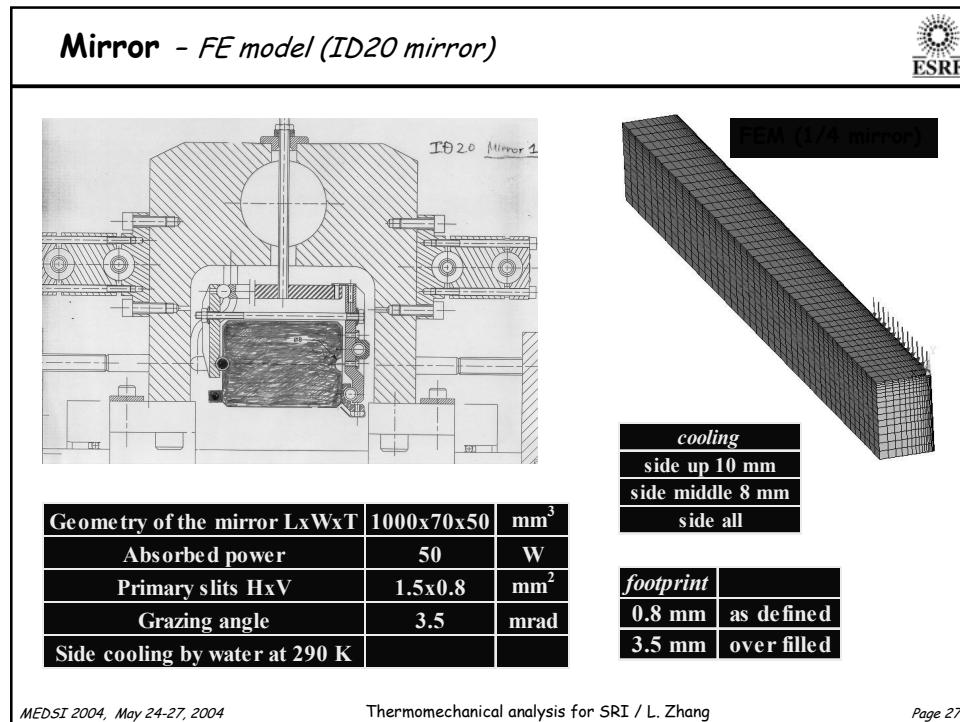
- Previous theoretical analysis is based on the mechanical beam theory.
- The X-ray mirror (1000x50x50) can be considered as a mechanical beam, but not the crystal monochromator (100x50x50)
- First order estimation for both mirror and monochromator:

$$\Delta\theta = \frac{\alpha}{k} \frac{P}{w} f(t, L, Bi, L_{bm}) \quad \text{or} \quad \Delta\theta \propto \frac{\alpha}{k} \frac{P}{w}$$

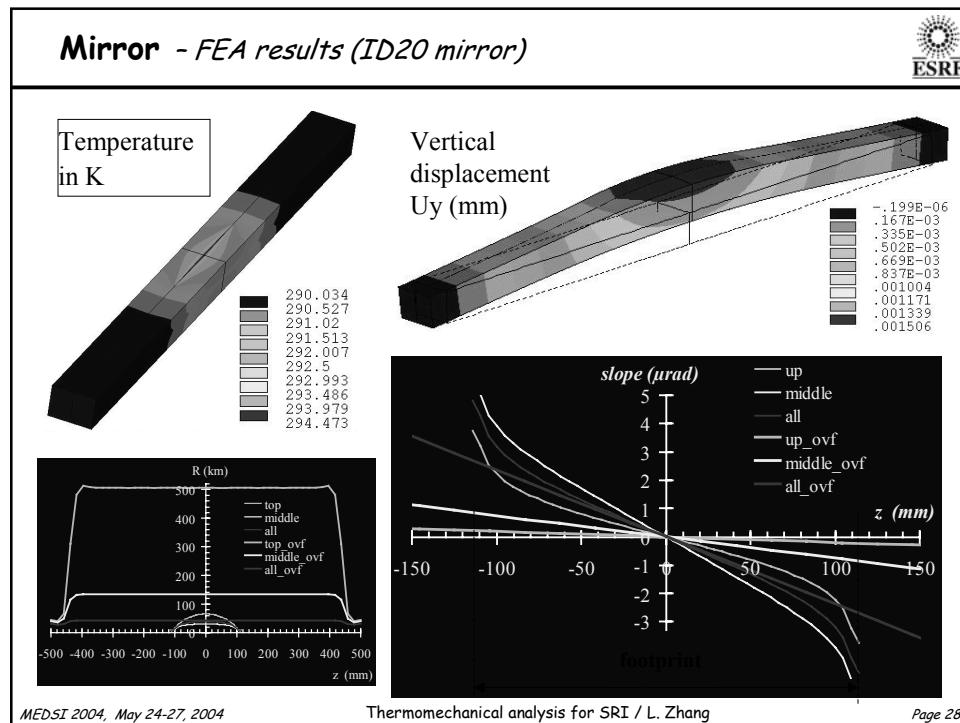
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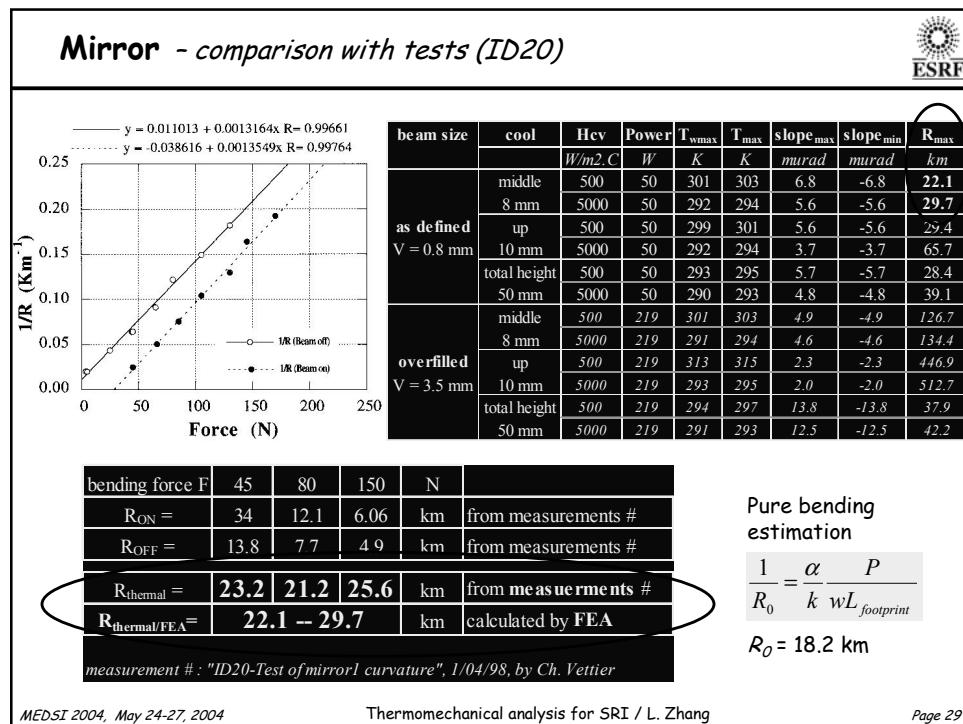
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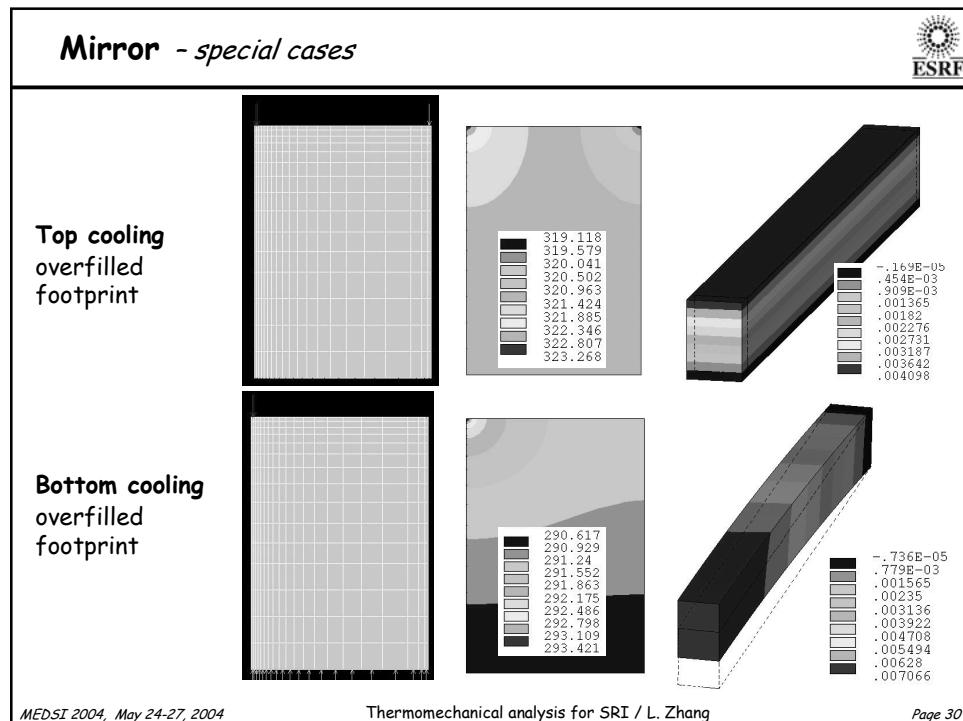


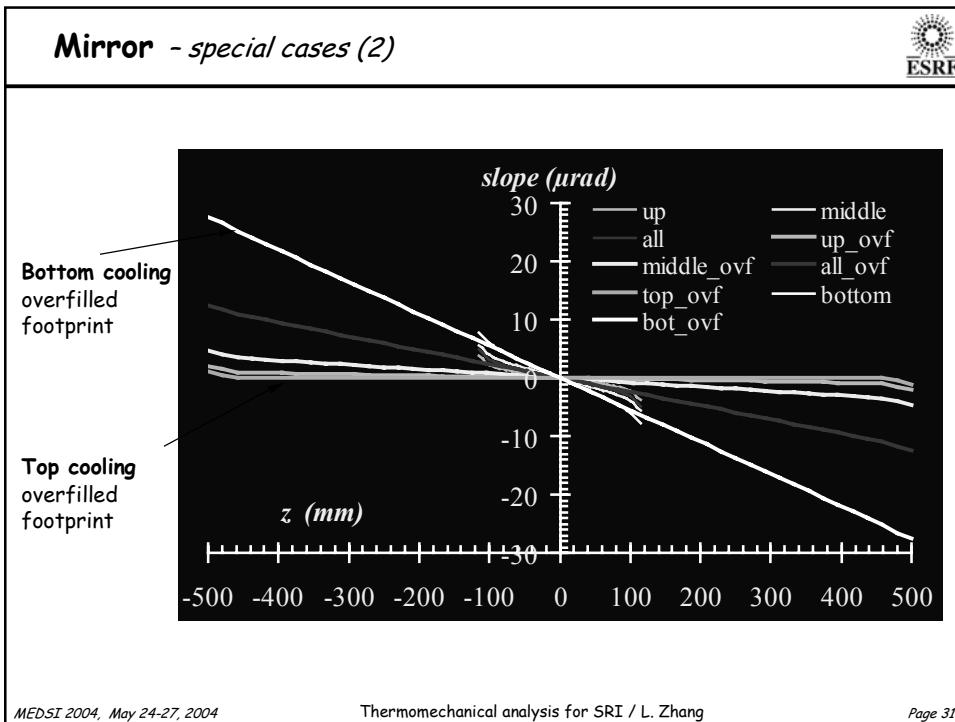
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Mirror - summary

α_{Si}	2.6	$\mu\text{m/m/K}$
k_{Si}	0.148	W/mm/K
cooling coefficient	0.005	W/m/K
mirror width w	70	mm
mirror length L	1000	mm
grazing angle	3.5	mrad
beam height (mm)	0.8	3.5
total power (W)	50	219
Peak-peak slope error (μrad)	within 228 mm	all mirror
cooling as defined	overfilled	overfilled
side, up 10 mm	7.48	0.45
side, middle 8 mm	11.14	1.72
side, total height	9.60	5.45
bottom cooling	15.64	12.61
top	0.00002	2.00
analytical $\alpha P/k/w$	12.55	54.96

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Monochromator crystal - FE model (ID9)

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- ID9 channel-cut Si crystal monochromator
- Liquid nitrogen cooling from 2 sides
- Beam size 10.35 mm (H) x 2.3 mm (V)
- Bragg angle 14°
- FE model based on CAD file
- Power distribution:

$$\frac{dP}{dV} = \frac{f_{Vabs} Pa_0}{\Delta t} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

- $\sigma_x = 8.114$ mm, $\sigma_y = 1.448$ mm
- Volume absorption coefficient:

$$f_{Vabs} = \frac{P_{abs} - \Delta t}{P_{incident}}$$

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Monochromator crystal - FEA results (ID9)

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Thermal slope error versus absorbed power in 4 different cooling coefficients

- Slope error independent of the cooling coefficient in the linear region → direct cooling (for high h) is not always necessary
- Slope error varies significantly with the cooling coefficient in the non-linear region

Slope ~ P curve can be divided in 3 regions:

- Linear region : $slope \sim P$
- Transition region : a local minimum
- Non-linear region : $slope \sim P^{4.6}$

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Monochromator crystal - Rocking width calculation



Rocking width calculation

- The rocking-curve width $FWHM_c$ can be calculated by

$$FWHM_c = \sqrt{(\theta_{th} + \theta_0)^2 + FWHM_{intr}^2}$$

- θ_{th} : FEA calculated thermal slope error
- θ_0 : initial deformation of the crystal due to mounting/fabrication-induced strain:

$$\theta_0 = \sqrt{(FWHM_{P=0})^2 - FWHM_{intr}^2} = 1.1 \text{ arcsec}$$

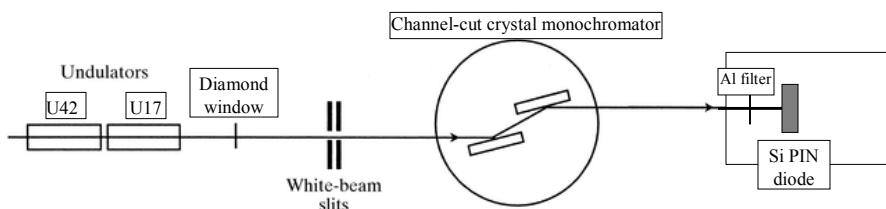
- $FWHM_{intr}$: intrinsic rocking-curve width [=0.67 arcsec for Si(333) at 24 keV]

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Monochromator crystal - test results (ID9)



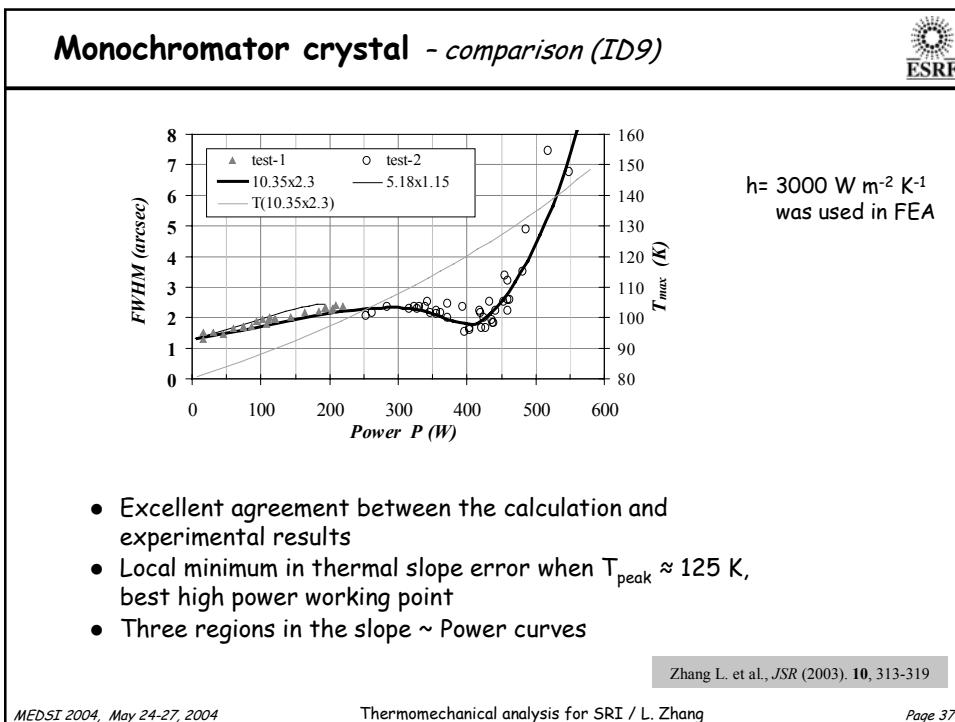
➤ Schematic diagram of the experimental set-up



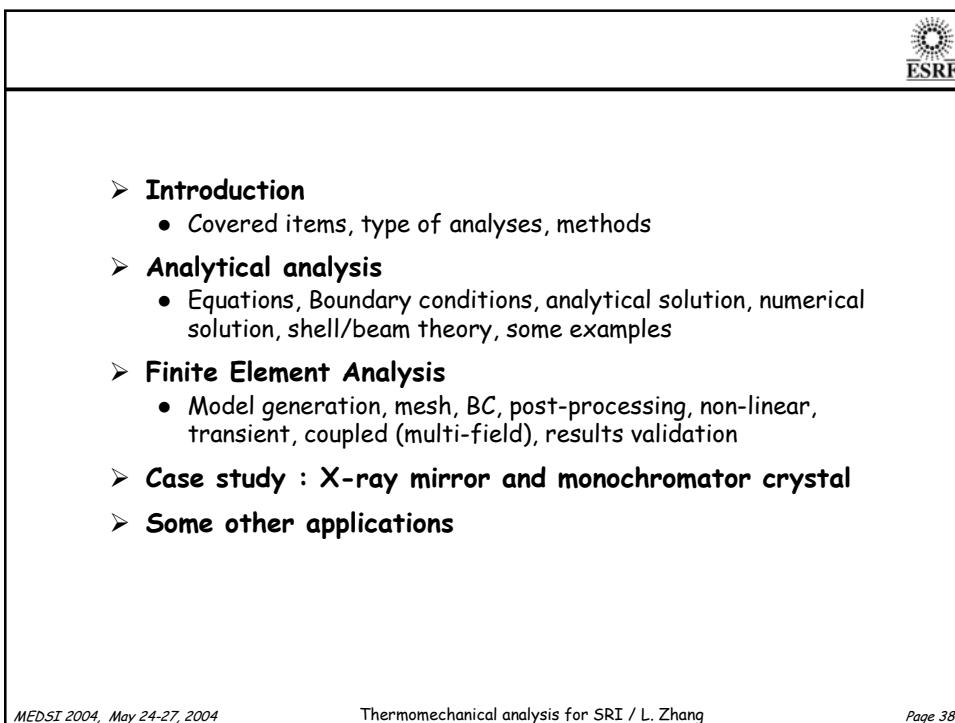
Measurement parameters

	First measurement	Second measurement
Undulators	U46	U46 & U17
Beam size (HxV) *	Varied	10.35 mm x 2.3 mm
Filter/window	0.5mm C + 0.5mm Be	0.4 mm diamond
Crystal Bragg angle	14.3°	14.3°

* Normal incidence at the crystal position



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Damping link for magnet-girder assembly

natural frequencies comparison

No	f_{TEST}	before tuning		after tuning	
		f_{FEM}	Δ	f_{FEM}	Δ
1	8.68	8.89	2.4%	8.64	-0.5%
2	11.74	11.64	-0.8%	11.75	0.1%
3	13.63	12.86	-5.6%	13.70	0.5%
4	22.33	22.47	0.6%	22.47	0.6%
5	26.29	26.45	0.6%	26.35	0.2%
6	27.82	27.17	-2.3%	27.14	-2.5%
7	32.18	31.58	-1.9%	31.48	-2.2%
8	32.30	33.12	2.5%	33.13	2.6%
9	34.85	36.39	4.4%	36.38	4.4%
10	39.49	38.29	-3.0%	38.28	-3.1%

➤ Elements: 3D solid, shell, beam
 ➤ Modal analysis vs Modal testing

- Natural frequencies and mode shapes
- ξ_i , stiffness of the 3 motorized jacks
- FE model validation

 ➤ Validated FEM → design optimization and simulation of dampers

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Damping link for magnet-girder assembly (2)

➤ Harmonic response analysis (HRA)

- 3 methods (MSUP, FULL, REDUCE)
- Response function U_y/F_y , Transfer function U_y/D_y
- Damping ratio
 - Initial system: MDAMP
 - Initial system + damper: DMPRAT= ξ_i for bandwidth $[f_{b,i}, f_{e,i}]$, $i=1,2,\dots$, using multi load steps (LSWRITE, LSOLVE)

➤ Modeling of the damper

- 3D solid (solid45)
 - Not effective for HRA
 - Independently to calculate the stiffness k
- Spring-damper (combin14)
 - Effective for HRA
 - In the assembly, $C=k h (2\pi f)^{-1}$

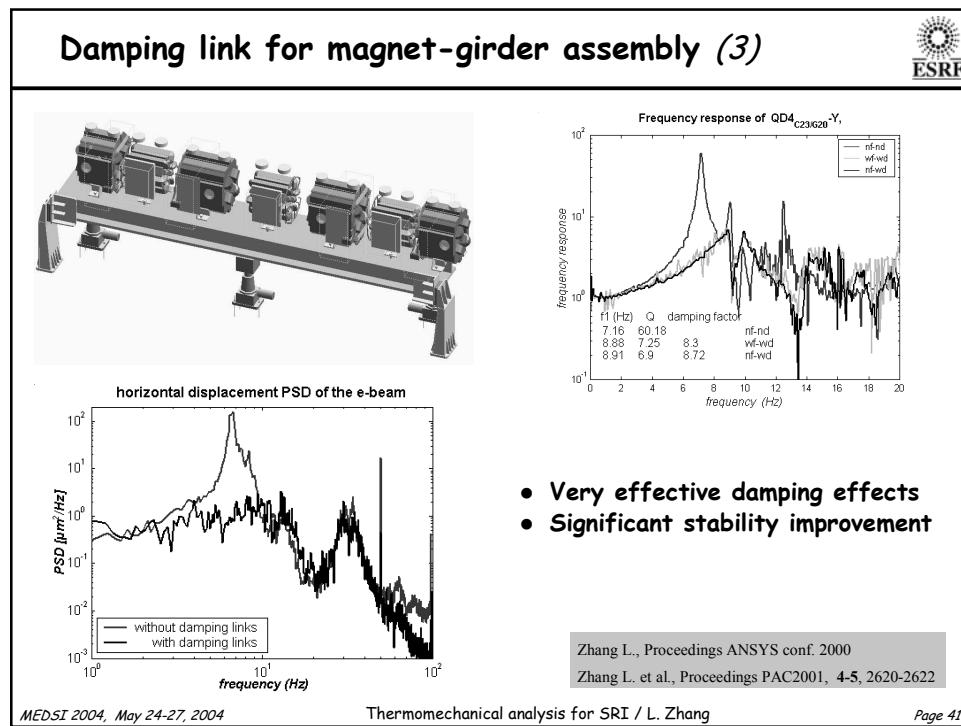
FrF_{Oryg10proto} in lateral direction: FEA and Test Results

Frequencies (Hz): 0, 5, 10, 15, 20, 25, 30, 35, 40

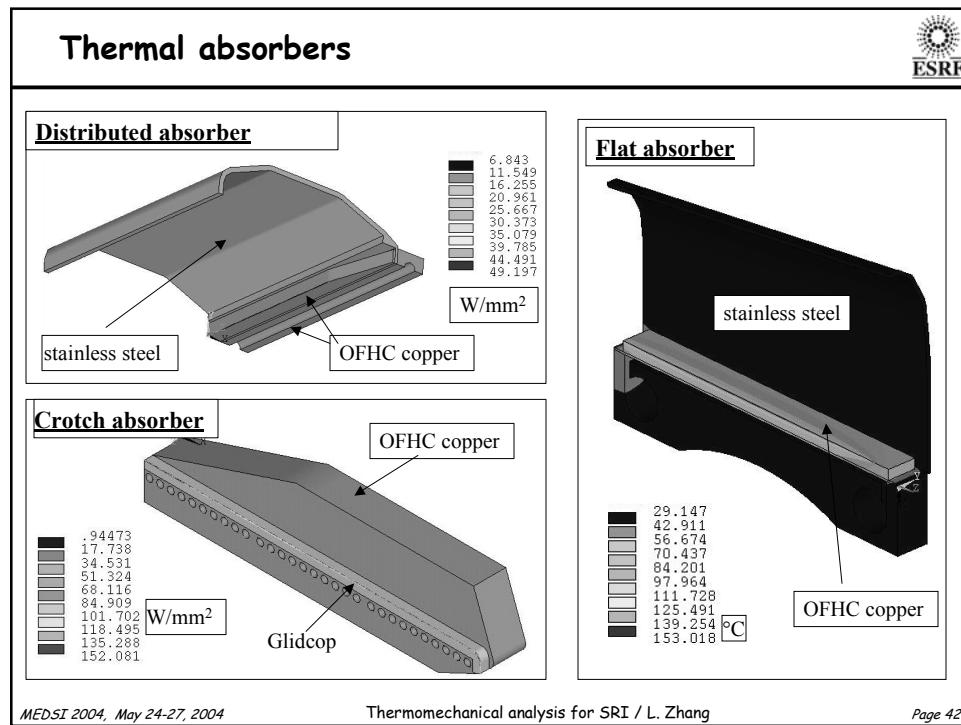
FrF (Nm/N): $10^0, 10^1, 10^2, 10^3, 10^4$

Legend: FEA_{MDAMP}, FEA_{DMPRAT=0.0256}, FEA_{DMPRAT=0.0256,0.0098}, FEA_{DMPRAT=0.0098}, Test

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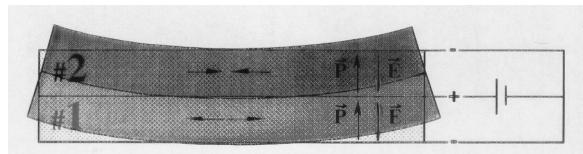
Thermal absorbers (2)



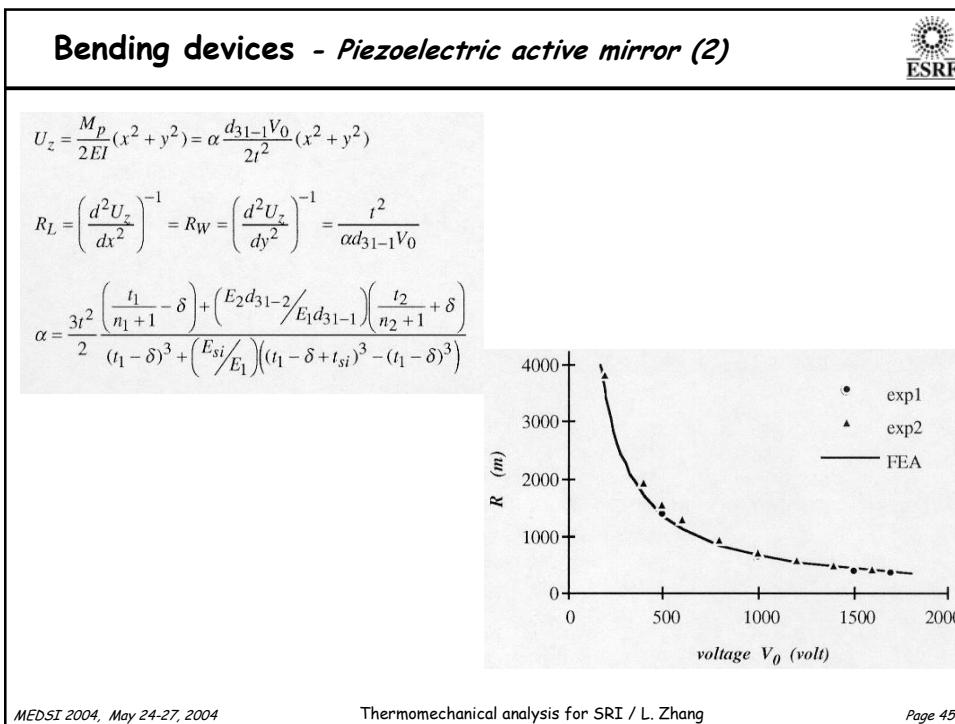
- **Conditions**
 - Copper (Glidcop or/and OFHC) or/and stainless steel
 - Total power ~ 10 kW, Power density ~ 300 W mm⁻²
 - Heat load concentration: $\sigma \sim 0.15$ mm, $L_{abs} \sim 180$ mm
 - Water cooled, correct heat transfer coefficient ($h \sim 10/20$ kW m⁻² C⁻¹)
- **Thermal analysis → temperature distribution → mechanical analysis → stress and deformation**
- **Difficulties due to small gaussian power distribution loading area**
 - Large number of elements: variable meshing, maximum reasonable capacity of the computers (*No of elements 2000 → 200 000 in 10 years*)
 - Delimitation of the parts, boundary conditions
 - Time consuming for heat flux loading: optimization of macros
- **FEA results validation**
 - Parameter studies: mesh, boundary conditions,...
 - Comparison with experiments

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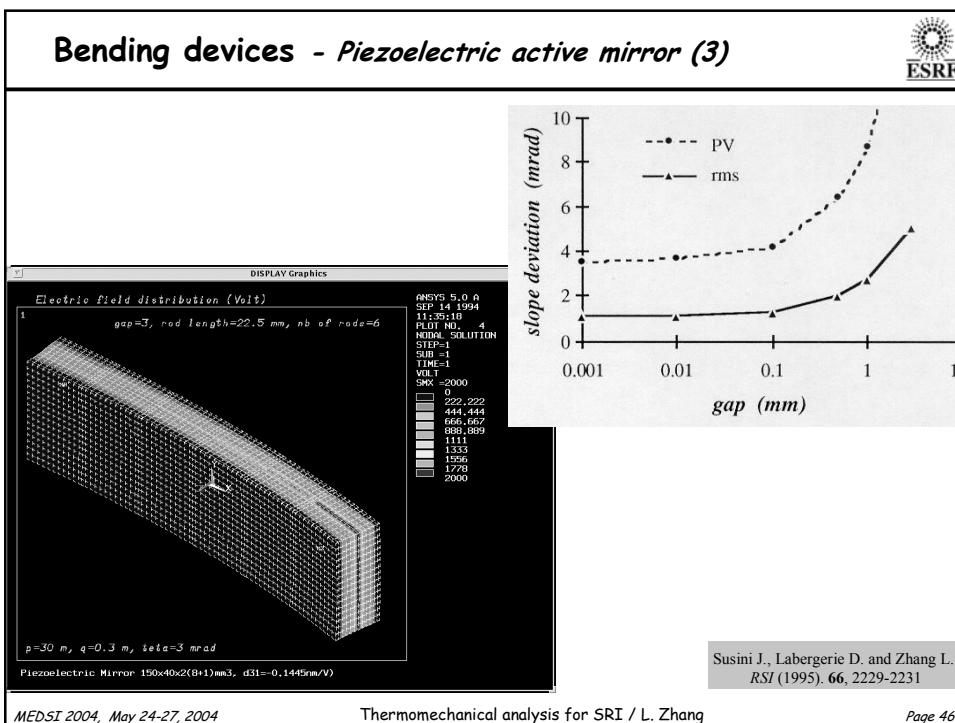
Bending devices - Piezoelectric active mirror

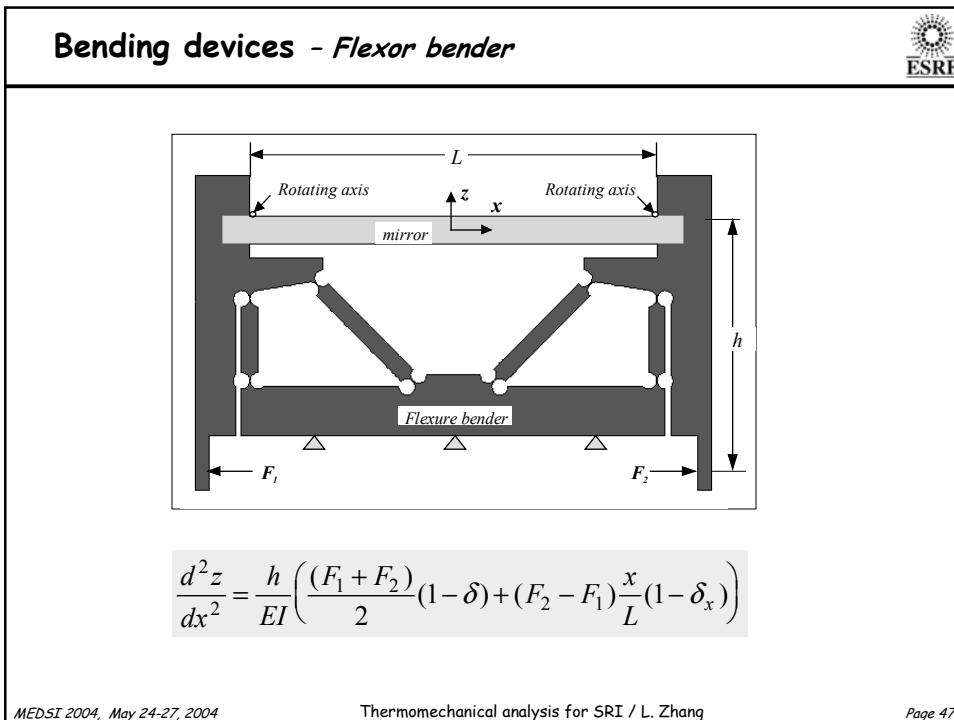


- **Piezoelectric bimorph: spherical shape**
- **Active mirror**
 - Spherical shape (mono-electrode)
 - Toroidal shape (multi-electrodes):
 - **Active:** variation of the radius of curvature by changing electric voltage
- **FEA key points**
 - electrode distribution, gap effects for a required shape
 - Piezoelectric matrix, elastic coefficient matrix

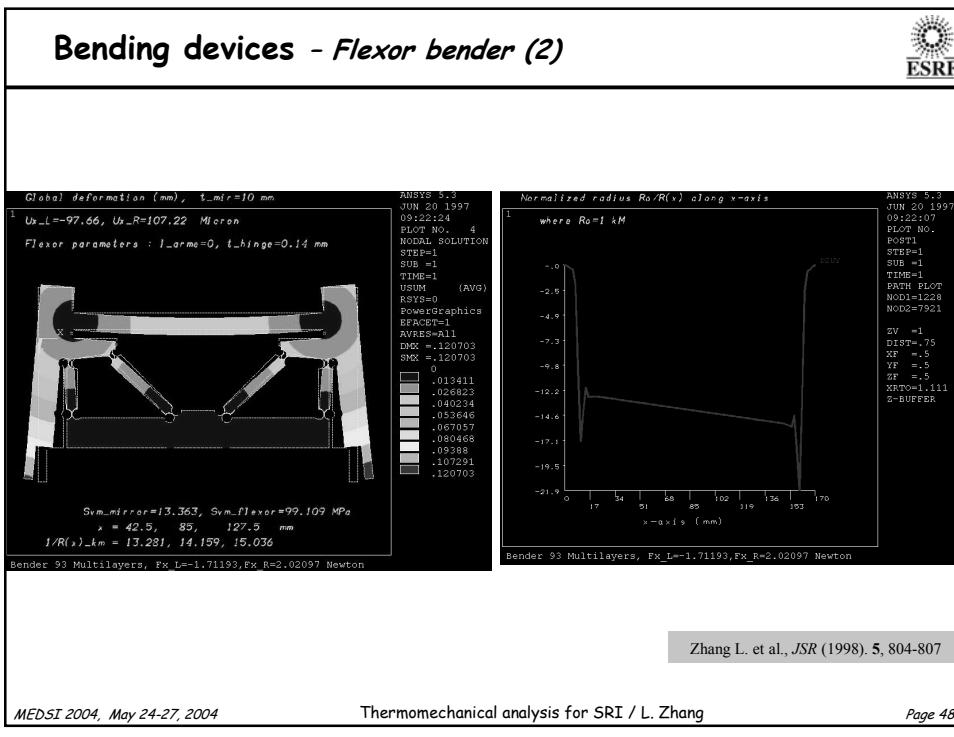


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Nano-structure in semiconductors



- **Quantum confinement in nm-scale semiconductor materials**
 - Nano-structures
 - Quantum well (electron motion restricted in 2D)
 - Quantum wire (electron motion restricted in 1D)
 - Quantum dot (electron motion restricted in 0D)
 - Semiconductor lasers for electronic and optoelectronic devices, better properties
- **High-resolution X-ray diffraction**
 - Characterization of geometric shape, composition and lattice strain
 - Imperfection of the diffraction pattern due to the strain field
 - Modeling by FEA → strain field

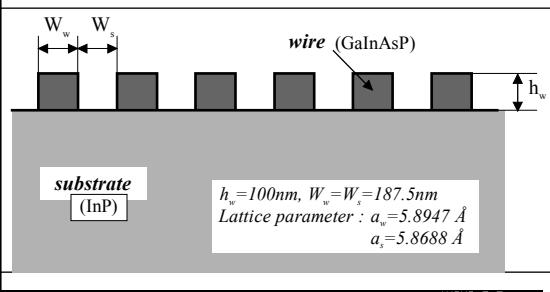
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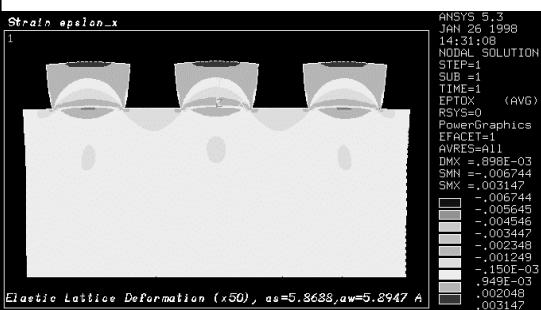
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Nano-structure in semiconductors (2)



- Differential lattice parameters ($\Delta a = 0.0259 \text{ \AA}$)
- The effects of differential lattice parameters simulated by thermal expansion effects
- Constant thermal strain in the wire is defined by :

$$\varepsilon_w = \alpha T = \frac{a_w - a_s}{a_s} = 0.00441$$



- The substrate: T_{ref}
- The wire: $T_w = T_{ref} + \varepsilon_w / \alpha$
- α is the thermal expansion coefficient, could be arbitrary value

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Planarity defect analysis of a Silicon wafer

Si Wafer, $\Phi 200$, thickness $t_{si} (\sim 0.7, 0.8 \text{ mm})$

Dielectric (ϵ_r) layer, thickness $t_{dielectric} (=0.2 \sim 0.5 \text{ mm})$

Electrode + - Electrode

Gripper

- TXRF technique → surface concentrations of the various elements, contamination analysis of $\Phi 200$ & 300mm semiconductor wafers
- High planarity requirement of the wafers

ANSYS Finite Element Model (ZOOM)
AUG 16 2001
17:11:46

Si wafer contaminant particle Gripper

ID27 Si wafer/chuck, t_q=0.2mm, W/2=49mm, gap=100μm, P=1, E=0.2MPa

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Planarity defect analysis of a Silicon wafer (2)

ANSYS Finite Element Model (ZOOM)
AUG 16 2001
17:11:46

Si wafer contaminant particle Gripper

ID27 Si wafer/chuck, t_q=0.2mm, W/2=49mm, gap=100μm, P=1, E=0.2MPa

vertical displacement (μm)

half contact length B_c

angular deformation (μrad)

maximum angular deformation $slope_{max}$

P

$$y = \frac{PL^4}{8E_{si}I}$$

- characteristic length L_c :

$$L_c = \left(\frac{8E_{si}IR_p}{P_{grip}} \right)^{0.25} = \left(\frac{2E_{si}t_{wafer}^3 R_p}{3P_{grip}} \right)^{0.25}$$

- The value of $L_c=10.6\text{mm}$ for
 - $P_{grip}=0.01 \text{ N/mm}^2 (\text{MPa})$
 - $t_{wafer}=0.7\text{mm}$
 - $R_p=5 \mu\text{m}$
 - $E_{si}=112.4 \text{ GPa}$
- Contact length $B_c=C*L_c$, coefficient C to be calculated by FEA
- FE model will be limited to $2*L_c$, instead of 200 or 300 mm

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Planarity defect analysis of a Silicon wafer (3)



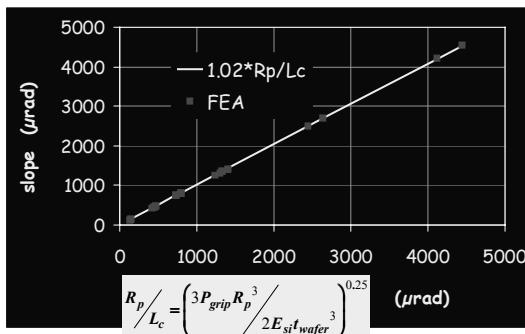
➤ From FEA:

- $C = 1.68 \rightarrow$ Contact length :

$$B_c = 1.68L_c = 1.52 \left(\frac{E_{si} t_{wafer}^3 R_p}{P_{grip}} \right)^{0.25}$$

- slope (angular deformation):

$$slope_{max} = 1.020 \frac{R_p}{L_c} = 1.129 \left(\frac{P_{grip} R_p^3}{E_{si} t_{wafer}^3} \right)^{0.25}$$



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Conclusion



- Thermomechanical analyses including analytical analysis and FEA have played and will continue to play an important role in
 - Instrument development
 - Design optimization and simulation
 - Various research
- Both analytical and FE analyses are complementary
- Analysis model has to be validated by experiments